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ABSTRACT

The purpose of this thesis is to study theoretically and experimentally the dynamics of an array of mutually coupled Van der Pol oscillators with applications to electric power generation and secure communications network.

The work is divided in four chapters.

In the first chapter we relate some general aspects of chaos synchronization and its potential applications in cryptography and secure communications.

In the second chapter, we perform the stability analysis for the synchronization of coupled oscillators. We use the Floquet theory to determine synchronization domain for two unidirectionally coupled Van der Pol oscillators as well as for a ring of four mutually coupled Van der Pol oscillators. The effect of parameter mismatch is also analyzed through the slightly different parameters between the oscillators.

In chapter 3, demonstrations of coding and decoding of messages as well as a selective secure communications scheme between various phones box are presented. Our secure scheme is able to take into account parameter mismatch between the transmitter and the receiver.

The last chapter focuses on the dynamics of network of loaded Van der Pol oscillators. The influence of coupling capacitance as well as the load quality factor in the electric power delivered in the load by the circuit is treated in this chapter. The condition to obtain maximum electric output power is also addressed.

Good agreement is found between analytical results and direct numerical simulations as well as experimental results.

RÉSUMÉ

RÉSUMÉ

Le but de cette thèse est d'étudier théoriquement et expérimentalement la dynamique d'un réseau d'oscillateurs de Van der Pol mutuellement couplés avec des applications comme générateur de tensions alternatives et comme réseau sécurisé des communications.

Ce travail comporte quatre chapitres.

Dans le premier chapitre nous rappelons quelques aspects généraux de la synchronisation du chaos et ses potentielles applications dans la cryptographie des données et la sécurisation des communications.

Dans le deuxième chapitre, nous utilisons la théorie de Floquet pour déterminer le domaine de synchronisation de deux oscillateurs de Van der Pol couplés de manière unidirectionnelle, puis d'un anneau de quatre oscillateurs mutuellement couplés. L'effet de disparité des paramètres est également analysé en considérant les oscillateurs ayant les paramètres légèrement différents.

Dans le chapitre 3, les démonstrations du codage et du décodage du message aussi bien qu'un schéma de communications sécurisées entre plusieurs interlocuteurs sont présentés. Notre schéma tient compte de la disparité de paramètres entre l'émetteur et les récepteurs.

Le dernier chapitre repose sur la dynamique d'un réseau d'oscillateurs de Van der Pol chargé. La condition d'obtention de la puissance électrique maximale est également étudiée.

Des bons accords sont obtenus entre les résultats théoriques et expérimentaux.

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